

Genetic Algorithm for Lattice Gauge Theory.

On SU(2) and U(1) on 4 dimensional lattice, how to hitchhike to thermal equilibrium state .

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Applying Genetic Algorithm for the Lattice Gauge Theory is formed to be an effective method to minimize the action of gauge field on a lattice. In 4 dimensions, the critical point and the Wilson loop behaviour of SU(2) lattice gauge theory as well as the phase transition of U(1) theory have been studied. The proper coding method has been developed in order to avoid the increase of necessary memory and the overload of calculation for Genetic Algorithm. How hichhikers toward equilibrium appear against kidnappers is clarified.

The goal of numerical simulation is to get the maximum (minimum) value of a given objective function parametrized by some set of independent variables. A genetic algorithm originated by J.H.Holland [1] is known that it is good at searching the maximum/minimum value under complicated situations. It bases its search on a set of point/individual of a searching space and on selection with respect to its fitness value, that is, the search of genetic algorithm is a global one rather than those of Metropolis and Heatbath method. Some severe extra works for coding a problem toward an appropriate form and treating some number of individuals at once had made its application wait until operating ability of computer system becomes adequate. By the recent development of the computer systems, its wide implementation becomes possible. However, it is still a subtle point that how to code a genome and how to design operations for the sake of good performance . We develop the coding of genomes in the SU(N) and U(N) lattice gauge theories and investigate to tune its searching parameters.

1. Algorithm and Coding

A configuration on a lattice is treated as a phenotype. Link variables on a site are taken from the link variable pool created in advance by the integer number index which is coded to a gene, a binary string. In this coding, the memory for genetic algorithm operation is able to be suppressed as follows; With the number of link variables in a link pool, 2^{L_p} , the lattice size, $2^{L_x+L_y+L_z+L_t}$, and the population size, 2^{pop_size} , the length of a

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genome becomes $L_p \times n_dim \times$ lattice size.

For instance, for 8 SU(2) lattice, a genome(a lattice) needs 282kb, and a population (32 lattices) needs about 9.2Mb. On the other hand, but for the genetic algorithm, the same size lattice needs 1.5 Mb and 32 lattices would need 48Mb. That is, a genetic operation needs only a quarter of memory for other methods.

Our algorithm has three genetic operations; besides ordinal ones of selection and recombination, a new one we call "the intergenerational conflict" is introduced to establish the thermal equilibrium. Furthermore as a local search, Metropolis updating applied to a link variable on a lattice is added, with which our combined algorithm behaves like a hybrid genetic algorithm.

This extraneous operation, the intergenerational conflict, is applied to the parent with a smaller action between a pair of parents' genomes and the offspring with a smaller action between the pair of offsprings produced from the parents. Offspring is passed to the next generation under the condition expressed using an ordinary Metropolis function,

$$\min\{1, \exp^{-\beta(S_o - S_p)}\} > \xi,$$

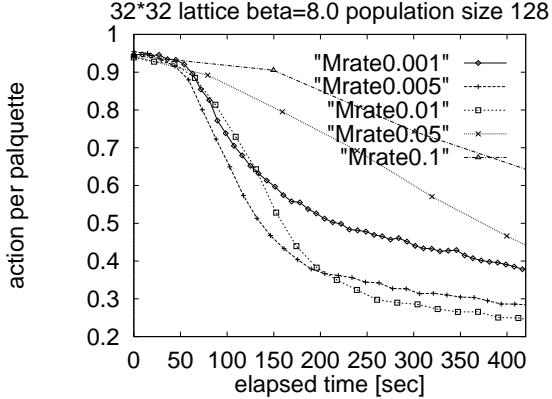
where

$$S = S[U] = \beta \sum_p \left(1 - \frac{1}{N} \text{Re} \text{Tr} U_p \right),$$

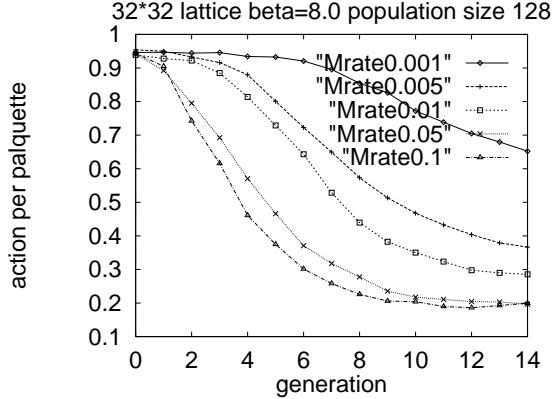
ξ an uniform random number.

Otherwise the parent is passed. Metropolis updating is applied to the survivor of the intergenerational conflict.

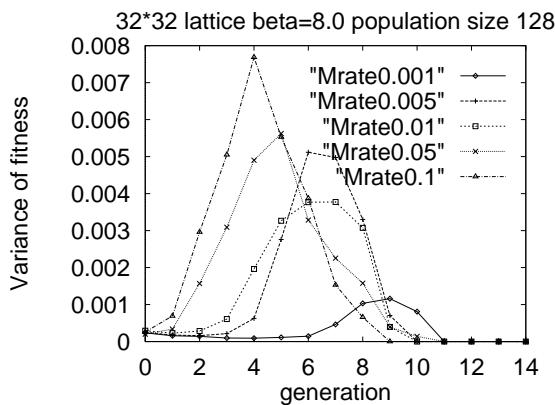
All genetic operations are coded in bit calculations. Arithmetic operation which needs more



The horizontal axis is computing time.



The horizontal axis is the number of generation.

Figure 1. Thermalizaion of Action per Plaquette 32 · 32 lattice $\beta = 8.0$, population size 128Figure 2. The variance of fitness, 32 · 32 lattice $\beta = 8.0$, population size 128

executed time is used for the calculation of the action. This coding can reduce the increasing of the executing time. For instance, genetic operations for 8^4 lattice with $\beta = 2$ occupies 17 percent of the executing time for one cycle, that is the increasing time caused by genetic operations is 15 percent of Metropolis method. This analysis conditions our genetic algorithm to an advantage against the usual Metropolis method.

2. The algorithm dynamics

To discuss the profitable condition, the dynamics of our algorithm on the statistical point of view is analized. For the procedure of Metropolis updating on a lattice is a random selection of a link \Rightarrow decoding information of gene \Rightarrow fixing the index of the link variable \Rightarrow updating the variable. This Metropolis updating makes configurations converge fast or slow and or not converge at all, because it work not only on the target lattice but also on the other lattice in the stack. This updating duplication makes two cases occur. First one is that if Metropolis updating works well even for another lattice configuration, the other lattice's action decreases. That is, its fitness value increases, and the lattice which is not the target of the Metropolis updating does hitchhike. Second one is the opposite case, that is, if Metropolis updating works well for the target lattice configuration but for the other lattice in the stack, its action increases that is its fitness value decreases. In this case, the target lattice kidnaps the other one. For the kidnapped lattice, the probability to be selected as a parent becomes low.

The behaviour of Hitchhiker and Kidnaper are clarified from the statistical value of the fitness. Five different Metropolis updating ratios, from 0.1 percents to 10 percents are set. From Fig. 1, the Metropolis updating ratio is found to de-

Table 1

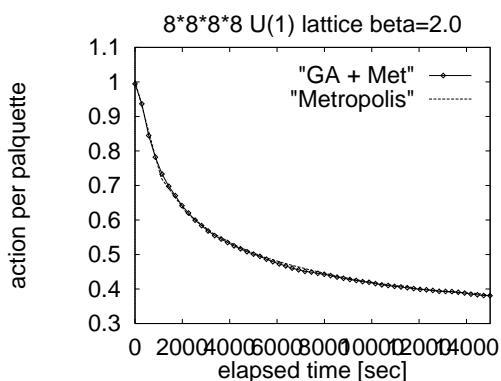
U(1) Action on 4 dimension

β	Metropolis	GA
1.0	0.538	0.538
1.5	0.370	0.369
2.0	0.298	0.292
3.0	0.237	0.235

Table 2

SU(2) Action on 4 dimension

β	Metropolis	GA
2.0	0.565	0.564
4.0	0.384	0.34

Figure 3. Thermalization of the U(1) action on 8^4 lattice $\beta = 2.0$, GA population size 16 and Metropolis method

pend strongly on the performance, so that the Metropolis updating ratio should be set 1 percent and not 5 percent nor 0.5 percent.

In Fig. 1 with generation, the effect of kidnappers and hitchhikers are appeared. There is not so clear difference between 10 percent line and 5 percent line. The slowing down of 10 percent line is caused by the kidnappers. On the other hand, the line of 1 percent and 5 percent fall rather fast, which shows there are some hitchhikers in the populations. From the variance of fitness, the populations which have some hitchhikers keep the variance at some level not so high and not so low. It means that some lattices might be kidnapped but other hitchhiking lattices accelerate its thermalization speed. It means the ratio of Metropolis updating has to be tuned. We set 0.01 for $\beta = 8.0$ and 0.005 for $\beta = 2.0$.

3. Result and Discussion

The physical values obtained by our algorithm are consistent with those given by Metropolis. See Tab. 1 and 2.

The mechanism Hitchhiker and Kidnaper is clarified from the statistical value of the fitness. Since the Metropolis updating ratio depends strongly on the performance, it should be set 1 percent not 5 percent nor 0.5 percent to $\beta = 2.0$.

Without any tuning parameters, a performance of our algorithm for U(1) on 4 dimensional lattice with 16 population size and $\beta = 2.0$, is same as that of Metropolis method. See Fig. 3.

For SU(2), on 2 dimensional lattice, our algorithm can get higher performance than Metropolis method [2]. On 4 dimensional lattice, however, it's performance is about the same as that of Metropolis. We have not tuned the parameter for hitchhikers on 4 dimensional lattice yet, but the performance is strongly dependent on the Metropolis updating ratio, because it occupies calculation time mainly.

Our work suggests that an algorithm with genetic operation does work in the case that the thermal equilibrium state is to be established.

The introduction of fermion must follow as the next stage, since our coding is good for SU(3) lattice gauge theory.

4. Acknowledgment

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